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MACHINING OF CORTICAL BONE: SIMULATIONS OF CHIP FORMATION MECHANICS USING METAL MACHINING MODELS

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This paper investigates chip formation in the machining of cortical bone and the application of isotropic elastic-plastic material models with a pressure dependent yield stress and a strain path dependent failure strain law to finite element calculations to predict observed behaviour. It is shown that a range of models can be created that result in segmented chip formations and a range of specific cutting forces similar to those observed experimentally. Results from the simulations provide an explanation for differences in the ratio of thrust to cutting forces observed between previous experimental studies, namely that the cutting tools used may have had different edge sharpness or degree of damage induced by the material removal process. Measurements of edge profiles from one of these studies support that explanation and emphasize the importance of tool toughness in maintaining efficient cutting of bone.

Keywords bone machining, chip formation, cutting edge breakdown, finite elements

INTRODUCTION

Material removal plays an important role in both orthopaedic processes and restorative dentistry. In both of these fields cutting is required to remove tissue and to prepare sites to accommodate an implant or an engineered biomaterial. In recognition that the surgical tools often used in these fields possess a single or multiple cutting edges with well-defined geometry, chip formation is an integral part of the material removal process. Therefore, an understanding of chip formation in the machining of hard tissues (i.e., dentin and bone) and the influence of tool geometry and/or edge condition on the material removal process is essential.

Cortical (load bearing) bone is a fibre re-enforced material, as are fibre re-enforced plastics. Previous finite element studies of bone machining
Machining of Cortical Bone

(Sugita et al., 2009a), and of machining fibre re-enforced plastics (Arola et al., 1997, 2002; Rao et al., 2007) have considered these to be structurally anisotropic materials and have simulated chip formation through fracture and development of a critical stress state. They have modelled the loading of a cutting edge against the shoulder of a work piece and have shown how the forces to initiate fracture vary with fibre re-enforcement direction. However they have not followed the cycles of chip formation with cut distance that is necessary, for example, to follow the build-up of temperature due to machining that is of major importance in surgery. The primary purpose of the study reported here is to assess the extent to which a metal machining finite element (FE) model, including a strain-accumulation damage law, can be applied to predict chip formation and forces in bone machining. It could be complementary to the fracture mechanics approach. Previously published experimental data on chip formation and forces is taken as a benchmark for the assessment.

Early experimental studies on machining of both human and bovine cortical bone, (Jacobs et al., 1974; Wiggins and Malkin, 1978) reported that cutting forces depend strongly on the orientation of the bone’s fibre re-enforcements, the osteons or laminae (Figure 1), relative to the tool cutting edge and the cutting direction. A more recently published study reported similar findings in the orthogonal cutting of fresh (hydrated) bovine cortical bone (Yeager et al., 2008). The largest forces occur when the osteons are perpendicular to both the cutting edge and cutting direction (referred to hereafter as the transverse orientation).

Figure 2(a) shows the dependence of specific cutting force (the cutting force $F_C$ per unit uncut chip area) on uncut chip thickness $h$ (from 0.05 to 0.15 mm) and tool rake angle, $\gamma$ (from $-30$ to $+30^\circ$, in steps of $10^\circ$) for the transverse orientation. The figure shows that specific cutting force reduces with uncut chip thickness (from 500–700 MPa at $h = 0.05$ mm to 250–400 MPa at $h = 0.15$ mm) for the positive rake tools. In contrast, the negative rake tools show a more scattered behaviour, but the data could be interpreted as being nearly independent of uncut chip thickness, in the range 500–800 MPa (the one value of 1000 MPa at $h = 0.1$ mm has a very large uncertainty of $\pm 200$ MPa).

Figure 2(b) shows the dependence of the ratio of thrust force $F_T$ to cutting force $F_C$ on rake angle. Within the scatter of results the behaviour is independent of uncut chip thickness. For negative rake tools (trend line AB), the ratio reduces as $\gamma$ increases from $-30$ to $-10^\circ$. The data is consistent with the resultant force being inclined to the normal to the rake face at a friction angle $\lambda = 17 \pm 10^\circ$; this is equivalent to an average friction coefficient over the rake face $\mu_{av} \approx 0.3 \pm 0.2$. However, for the positive rake tools (trend line CD), $F_T/F_C$ is independent of $\gamma$, with a value in the range 1.0 to
FIGURE 1 Schematic diagram describing the terminology used in the orthogonal machining of bone. The O axis indicates the direction of the osteons and in (a), (b), and (c) the cutting velocity vector is \( V_c \) and the osteon orientation is O. (a), (b), (c) are the parallel, across and transverse directions in Jacobs et al. (1974) and the [0,0], [0,90] and [90,0] modes in Yeager et al. (2008). (c) is the orientation considered in this paper.

FIGURE 2 Dependence of (a) specific cutting force on uncut chip thickness and (b) ratio of thrust to cutting force dependence on rake angle, in machining cortical bone. The plots are based on Yeager et al., 2008 and Yeager, 2006. The cutting speed was \( V_c \approx 20 \text{ m/min} \) and width of cut 2 to 3 mm. The tools were uncoated carbide (C2 grade: WC, 6% Co).
1.2. This change in behaviour from negative to positive rake tools occurs as $\gamma$ increases from $-10$ to $+10^\circ$. Particularly as $\gamma$ approaches $+30^\circ$, $\mu_{av}$ increases above 3.0. How this may be explained is returned to later.

Similar specific cutting force variations have been found by other researchers. Wiggins and Malkin (1978) planed bovine cortical bone at $v_c \approx 0.5 \text{ m/min}$, $h$ from 0.01 to 0.4 mm, with high speed steel tools having $\gamma = -30$ to $+40^\circ$. For the positive rake tools, specific forces for machining in the transverse orientation reduced from 400–500 MPa at $h = 0.05$ mm to 250 – 400 MPa at $h = 0.15$ mm, matching the behaviour in Figure 2(a); the specific forces underwent further reduction to $\approx 100$ MPa at $h = 0.4$ mm. Negative rake tool data was not presented in so much detail but specific forces were less dependent on $h$, with values from 600 to 800 MPa in the range of $h$ from 0.05 to 0.15 mm. Wiggins and Malkin also presented data for machining human cortical bone, and for the transverse orientation results were almost identical to those for bovine bone.

Further results have been reported by Itoh et al. (1983). They planed both porcine and human cortical bone, with $v_c = 1.4 \text{ m/min}$, $h$ from 0.2 to 0.5 mm, using high speed steel and surgical stainless steel tools, with $\gamma$ ranging from $-5$ to $+20^\circ$. They found specific forces reducing from 200–250 MPa at $h = 0.2$ mm to 100–150 MPa at $h = 0.5$ mm, for bone of both species machined in the transverse orientation. That response follows the trends of Wiggins and Malkin at large $h$.

Other more recent studies confirm the reduction of specific force with increasing $h$, but report different cutting forces. In very slow cutting tests ($v_c = 0.012 \text{ m/min}$), with single crystal diamond tools ($\gamma$ from $+5$ to $+30^\circ$) Mitsuishi et al. (2005) reported specific forces for transverse cutting of human bone from 500 to 200 MPa at $h$ of only 0.002 to 0.02 mm. The mechanical properties of bone are strain-rate dependent. For example, the compressive strength of fresh bovine bone increased by $\approx 50\%$, from 250 to 365 MPa, as strain-rate increased from 1 to 1500/s (Reilly and Burstein, 1974). Thus, the lower specific forces from Mitsuishi et al.’s work might be argued to be a result of the lower strain-rates that would have occurred in their experiments. However similarly low values (100 to 400 MPa) have been reported at $v_c = 210 \text{ m/min}$ for bovine cortical bone with $h$ from 0.02 to 0.1 mm and using high speed steel tools ($\gamma = 0$ to $40^\circ$) (Plaskos et al., 2003). Of note, specific cutting force depends on the density of cortical bone. In milling experiments ($v_c \approx 30 \text{ m/min}$, $h_{max} = 0.03$ mm) on porcine cortical bone specific cutting forces varied from 170 to 250 MPa as sample density was changed from 1.8 to 2.0 g/cm$^3$ (Mitsuishi et al., 2004).

In contrast to the specific force variations shown in Figure 2(a), the variations of $F_T/F_C$ with rake angle (Figure 2(b)) differ in some respects (for positive rake tools) from those observed by other researchers. For
h = 0.05 to 0.15 mm, Wiggins and Malkin (1978) observed $F_T/F_C = 0.7$ to 0.8 for $\gamma = -10^\circ$, falling to 0.35 to 0.55 at $\gamma = +10^\circ$, following the negative rake angle trend seen in Figure 2(b). These values are equivalent to friction angle $\lambda = 30 \pm 5^\circ$ and $\mu_{av} \approx 0.6 \pm 0.1$ over that rake angle range. But as $\lambda$ increased to $+40^\circ$, $F_T/F_C$ remained at the value of 0.35 to 0.45 (trend line C/D), constant but low compared to 1.0 to 1.2 (trend line CD).

Itoh et al. (1983) commented that high speed steel tools gave larger $F_T/F_C$ values than stainless steel tools. For stainless steel tools $F_T/F_C$ varied with $\gamma$ to give $\mu_{av} = 0.5 \pm 0.1$ over their whole experimental range, i.e., $F_T/F_C \approx 0.2$ at $\gamma = +20^\circ$ (demarcated C//D// in Figure 2(b)). Thus, the experimental results follow two separate trends, one at negative rake angles represented by the line AB in Figure 2(b) in which the resultant force direction remains constant relative to the normal to the rake face as $\gamma$ changes, and the other at positive rake angles represented by lines CD in which $F_T/F_C$ remains constant at a value unique to the different researchers. An understanding of the cause for these differences could provide insight that facilitates the development of improved tools for machining of bone.

As presented earlier, the primary objective of the study reported here was to assess the extent to which a metal machining finite element (FE) model, including a strain-accumulation damage law, could be applied to predict chip formation and forces in bone machining. However, during the course of study it emerged that the modelling could explain differences in reported forces between previous experimental studies in terms of possibly different cutting edge conditions in these studies. Thus, the efforts were modified to include an examination of the effects from tool edge condition on the cutting mechanics and chip formation. Further, a justification for the study was given as creating an ability to determine temperatures in bone machining. Some preliminary temperature results are also presented.

**MODELS AND SIMULATIONS**

One of the prerequisites for modelling of bone machining is having appropriate bone material flow stress and failure information. The dependence of compressive strength on strain-rate has already been mentioned. Low strain-rate data for fresh cortical bone, extracted from Reilly and Burstein (1974) is summarised in Table 1. In compressive conditions, strengths of bovine bone are $\approx 50\%$ larger than human bone. When the principal stress axis is aligned parallel to the osteon direction, strengths are $\approx 50\%$ greater than when the principal stress direction is perpendicular to the osteon direction. In tension (less relevant to machining), strengths are lower, particularly when testing perpendicular to the osteon direction but there is less difference between bovine and human bone.
Where data are available, strains to failure are typically \( \approx 1 \) to 2\% or less. Bone is clearly a brittle material relative to ductile metals. However Wiggins and Malkin (1978) estimated an apparent toughness of bone from their machining tests (they also presented quick-stop pictures of the segmented chip formation), by dividing the energy expended in cutting by the area of newly formed surface of segmented chips. Their value of \( 12 \, \text{kJ/m}^2 \) is of the same order of magnitude as for fibre reinforced plastics, considerably larger than for cast iron (0.2 to 3 kJ/m\(^2\)) and much larger than for extremely brittle materials such as ceramics and glass (<0.1 kJ/m\(^2\)) (Ashby and Jones, 1980). These relative values support the use of a plastic strain accumulation damage law, coupled with a plasticity analysis, for modelling chip formation in bone machining.

**Material and Friction Modelling**

In this investigation the commercial metal machining FE software AdvantEdge-2D\textsuperscript{TM} was used. The relations between stresses and strain increments in this software are isotropic elastic-plastic with the capability to include features of the flow and failure behaviour of materials with limited ductility. It has two ways to achieve this behaviour, either together or separately. One approach is to include a pressure dependence of the flow stress, in the manner of a Drucker-Prager yield criterion (Drucker and Prager, 1952), as

\[
\bar{\sigma} = f(\bar{\varepsilon}, \dot{\varepsilon}, T) + a \rho
\]

where \( \bar{\sigma}, \bar{\varepsilon}, \dot{\varepsilon}, T \) are equivalent stress, plastic strain and strain-rate respectively, with \( T \) the temperature. Also, \( \rho \) is the hydrostatic pressure and \( a \) is the coefficient of pressure dependence. The other approach is to include a path dependent damage accumulation and failure law. An element loses its stiffness when its damage fraction \( D \) reaches 1.0, as defined by

\[
D = \sum_{i=1}^{n} \frac{d\bar{\varepsilon}_i}{\bar{\varepsilon}_{f,i}}
\]
For D > 1, although stiffness becomes zero, there is no element and/or node elimination, so a crack does not open up. Rather, large displacements are triggered across the element concerned. Chip fragmentation may occur but it is a result of the action of re-meshing routines in the software rather than a consequence of a physical crack propagation mechanism. In equation 2 \( d\bar{\varepsilon}_i \) is the plastic strain increment in time step \( i \) and \( \bar{\varepsilon}_{f,i} \) is the strain to failure in the stress-state of time step \( i \). \( \bar{\varepsilon}_{f,i} \) is both material and hydrostatic stress dependent in a Johnson-Cook type manner (Johnson and Cook, 1985) as shown in equation 3a (from the software’s user manual, but with changed notation) and Figure 3.

\[
\frac{\bar{\varepsilon}_{f,i}}{\bar{\varepsilon}_{f,ref}} = \exp \left[ 1.5C_p \left\{ \left( \frac{\bar{\varepsilon}_c}{\bar{\varepsilon}_f} \right)_i - \left( \frac{\bar{\varepsilon}_t}{\bar{\varepsilon}_f} \right)_{ref} \right\} \right]
\] (3a)

\[
C_p = \frac{1}{\sqrt{3}} \ln \left( \frac{\bar{\varepsilon}_c}{\bar{\varepsilon}_t} \right)
\] (3b)

\[
\bar{\varepsilon}_{f,ref} = \sum_{j=0}^{5} d_j T^j
\] (3c)

The quantity \( p/\bar{\sigma} \) is the ratio of hydrostatic pressure to equivalent stress. \( C_p \) (equation 3b) depends on the ratio of the material’s equivalent strains to failure \( \bar{\varepsilon}_c \) and \( \bar{\varepsilon}_t \) in simple compression and tension conditions. \( \bar{\varepsilon}_{f,ref} \) is the

![](image)

**FIGURE 3** Failure strain relative to reference failure strain dependence on stress state \( p/\bar{\sigma} \) and \( C_p \).
material’s equivalent strain to failure at the reference stress state \((p/\bar{\sigma})_{\text{ref}}\). It can be made temperature dependent through the polynomial expansion equation 3c. In the software \((p/\bar{\sigma})_{\text{ref}}=1/\sqrt{3}\), as shown in Figure 3. It has particular significance for metal machining. It is the stress state that would exist at the primary shear plane if the hydrostatic pressure there were equal to the shear flow stress.

The friction model utilized in the software is as described in equation 4:

\[ \tau = \min\left(\mu \sigma_n, \frac{\bar{\sigma}}{\sqrt{3}}\right) \tag{4} \]

with \(\tau\) and \(\sigma_n\) the friction and normal contact stresses and \(\mu\) the sliding friction coefficient. For a sufficiently large value of \(\mu\) (typically 1.0 or more), friction stress becomes limited by the shear flow stress \(\bar{\sigma}/\sqrt{3}\). The direction \(\lambda\) of the resultant cutting force is then not the same as \(\tan^{-1}\mu\).

**Material Properties and Friction Coefficients**

It was chosen to model the strain, strain rate and temperature dependence of flow stress of bovine cortical bone in a simple manner (within the constraints imposed by the software), as the main purpose of the work was to explore introducing a damage law to the modelling. Equation 5a gives the general power law form of flow stress dependence on strain, strain-rate and temperature that is available within the software. Strain and strain-rate hardening are modelled through the values of \(\sigma_0\), \(\varepsilon_0\), n and m. Thermal softening is described by \(\Theta(T)\). Equation 5b gives the particular form used in the present work. Temperature dependence is ignored as, to avoid bone necrosis (thermal damage), temperature must be limited to beneath \(\approx 55^\circ\text{C}\), as reviewed by (Karmani, 2006). With the chosen strain and strain-rate hardening values, flow stress varies from 200 to 270 MPa as strain increases from 0 to 1.0 and strain-rate increases from 1 to \(10^4/s\). These values are similar to the expected range of values from compression testing of bovine bone along the osteon direction as reviewed earlier (Table 1).

\[ \bar{\sigma} = \sigma_0 \left(1 + \frac{\varepsilon}{\varepsilon_0}\right)^{1/n} \left(1 + \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{1/m} \Theta(T) \tag{5a} \]

\[ \bar{\sigma}(\text{MPa}) = 200 \left(1 + \frac{\bar{\varepsilon}}{0.009}\right)^{0.022} \left(1 + \dot{\varepsilon}\right)^{0.02} \tag{5b} \]

Preliminary simulations indicated that a range of chip formations from ductile to segmented (discontinuous) could be obtained from choosing a (equation 1) in the range 0.3 to 0.6 and \(C_p\) (equation 3b) between 0 and
1.0 (0.5 is associated with a ratio of compressive to tensile failure strain of 2.4). Strain to failure in the reference stress state was taken to be temperature independent for the same reason as was flow stress. Values of $d_0$ (equation 3c) in the range 0.3 to 1.0 were found to be associated with qualitative changes in chip formation. Table 2 lists the combinations of values that were used for the main series of simulations. The final (ranking) row relates to results and will be considered later.

Simulations were carried out at two levels of friction coefficient, $\mu = 0.2$ and 1.0. The lower limit ($\mu = 0.2$) is from experimental studies, whereas $\mu = 1.0$ is a high value at which friction stress becomes limited by shear flow stress. Table 3 records the physical properties assigned to the bone and tool materials. Bone thermal properties were taken from Davidson and James (2003). Based on preliminary simulations neither increasing Young’s modulus of bone to 20 GPa nor reducing Poisson’s ratio to 0.2 influenced results significantly. The tool properties are those of a K-grade carbide to match the experimental test conditions in Yeager et al. (2008).

**Machining Variables and Simulation Outputs**

Simulations were carried out at tool rake angles $\gamma$ from $-30$ to $+30^\circ$ and uncut chip thicknesses $h$ of 0.05, 0.1 and 0.2 mm, which spanned the experimental conditions being simulated. Cutting speed was set at 20 m/min, rounded up from the experimental value of 18.6 m/min. Tools were given a range of cutting edge radii $r_b$ from 10 to 40 $\mu$m.

Basic outputs from the simulations were the chip shapes and variations with time of the cutting $F_C$ and thrust $F_T$ forces per unit cutting edge engagement length. Fortran programs were written to calculate the time-averaged cutting and thrust forces from the software’s output files. Specific forces (forces per unit uncut chip area) and force ratios $F_T/F_C$ were also calculated. Chip and work surface temperature fields were also obtained. The simulations were carried out with the initial tool and work material temperatures set at 20$^\circ$C (the experimental data was from room temperature machining) and assuming no heat losses from the free surfaces.

**TABLE 2** Combinations of $C_p$, $a$, $d_0$ Chosen for the Simulations

<table>
<thead>
<tr>
<th>$C_p$</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>1.0</th>
<th>1.0</th>
<th>1.0</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
<td>0.3</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$d_0$</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
<td>0.9</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Rank*</td>
<td>9</td>
<td>6</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>3</td>
<td>13</td>
<td>11</td>
<td>2</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

*Ductility ranking (1 most ductile) from simulation results, Figure 4.
RESULTS OF MODELLING

Simulation Results ($h = 0.1 \text{ mm}$)

**Chip Formation and Specific Cutting Force Dependencies** Among all the investigated variables it was found that the damage model most strongly affects the specific cutting force and chip formation. Figure 4 shows the range of time-averaged specific cutting forces obtained from simulations conducted with all the model materials considered (Table 2) for $\gamma = 30^\circ$, $r_\beta = 20 \mu m$ and $\mu = 1.0$. The materials have been placed along the x-axis in order of reducing specific cutting force with the relation between rank and material model as recorded in Table 2. Overall, there is a 7-fold range of force over the material models considered. Figure 4 also shows the differences in chip formation and time variation of specific cutting force with changing ductility for four selected examples. They demonstrate the increasing unsteadiness of flow and forces as the apparent ductility of bone decreases.

Figure 5 shows the influence of friction coefficient on specific cutting force and chip formation, for $\gamma = 0^\circ$ and $r_\beta = 20 \mu m$. Instead of showing time varying forces as in Figure 4, it shows the influence of friction coefficient on chip formation. For each pair of views, the chip formed with $\mu = 0.2$ is above that formed with $\mu = 1.0$. As might be expected, a lower friction coefficient results in lower specific cutting forces, but the changes in force are small relative to those that can be obtained by varying the material damage model. The lower degree of friction causes the chips to be more curled and this leads (at least in the simulations) to more chip fragmentation.

Figure 6 shows the influence of cutting edge radius on specific cutting force and chip formation for $\gamma = -30^\circ$, $\mu = 0.2$ and $r_\beta = 10$ and $40 \mu m$. At least in the range studied ($r_\beta/h < 0.4$), cutting edge radius has only a small influence on cutting forces and negligible influence on chip formation. For this reason only one view of chip formation is shown for each of the four chosen damage model examples.

As evident from Figures 4 to 6, the material model is the primary factor contributing to the simulated specific cutting force. Comparisons between these figures show that rake angle has an intermediate contribution, while the friction coefficient and cutting edge radius have almost no influence.

<table>
<thead>
<tr>
<th>Property</th>
<th>Bone</th>
<th>Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (GPa)</td>
<td>10</td>
<td>–</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.4</td>
<td>–</td>
</tr>
<tr>
<td>Thermal conductivity (W/mK)</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>Specific heat (J/kgK)</td>
<td>1400</td>
<td>250</td>
</tr>
<tr>
<td>Density (kg/m$^3$)</td>
<td>2000</td>
<td>14,000</td>
</tr>
</tbody>
</table>
The small effect of friction and edge radius is explicitly shown in Figure 7(a) for $\gamma = +30^\circ$. In particular, the influence of friction decreases with reduction in the material model ductility, as the strain in the chip becomes limited by ductility rather than altered by friction coefficient. Figure 7(b) shows the range of specific forces from all the simulations, which decrease in magnitude with increasing rake angle. Note that the auxiliary axis shows the dimensionless specific cutting force, which is the specific cutting force normalized by the yield stress of the material (i.e., 200 MPa at the low strain and strain rate from equation 5).

The ranges of specific forces from 800 to 300 MPa ($\gamma = -30^\circ$) and 500 to 100 MPa ($\gamma = +30^\circ$) in Figure 7(b) overlap the experimental ranges evident in Figure 1(a). However the most and least ductile material models are not realistic. Comparison of the chip formations in Figures 4–6 with

FIGURE 4 The range of specific cutting forces and quality of chip formation, $h = 0.1$ mm, $\gamma = +30^\circ$, $r_p = 20$ mm, $\mu = 1.0$. 

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the quick-stop images in Wiggins and Malkin (1978) shows that the material models ranked 4 and 9 give realistic chip segmentation. The reduced specific cutting force ranges for models 4 to 9 are $\approx 600$ to $350$ MPa ($\gamma = -30^\circ$) and $\approx 450$ to $200$ MPa ($\gamma = +30^\circ$), which are slightly below the experimental ranges. It seems possible that choice of material ductility model in the range 4 to 9, coupled with adjustment of $\sigma_0$ (equation 5) to near 200 MPa will allow machining of cortical bone to be simulated by the software. This is considered further in the discussion section.

**Resultant Force Direction.** In contrast to the primary dependence of specific force on material model and rake angle, the main variables influencing the ratio ($F_T/F_C$) are the cutting edge radius and the rake angle.
Figure 8(a) shows the \((F_T/F_C)\) data from the \(\gamma = +30^\circ\) simulations. There is a small influence of friction coefficient when \(r_b = 10\) \(\mu m\) and there is a tendency for \((F_T/F_C)\) to be lower for the most brittle material model than for the others. Yet, the dominant view is the increase of \((F_T/F_C)\) with increasing cutting edge radius. Figure 8(b) compares the behaviour at \(\gamma = -30^\circ\) with that at \(\gamma = +30^\circ\) over the range of material models and in terms of the cutting edge radius; the \(\gamma = +30^\circ\) data are re-plotted from Figure 8(a). By contrast, for \(\gamma = -30^\circ\), \(F_T/F_C\) is in the range 0.8 to 1.3 and almost independent of cutting edge radius.

**Figure 6** The range of specific cutting forces and quality of chip formation, \(h = 0.1\) mm, \(\gamma = -30^\circ\), \(r_b = 10\) and 40 \(\mu m\), \(\mu = 0.2\) (chip formation independent of \(r_b\)).

**Figure 7** Specific cutting force dependence on (a) material model, \(r_p\) and \(\mu, \gamma = +30^\circ\); (b) rake angle, for all the simulations (material models 2, 4, 9, 11).
The experimental results for $\gamma = -30^\circ$ (Figure 2(b), $F_T/F_C = 0.8$ to $1.0$) are within the range obtained from the simulations. The unique results indicated by CD, C/D, C/D/ for $\gamma = +30^\circ$ in Figure 2(b) could be reconciled with the simulations if the experiments associated with them were with tools of different edge radius, i.e., $>40\,\mu m$ for the results CD, $\approx 20$ to $40\,\mu m$ for the results C/D/ and $<20\,\mu m$ for the results C/D/. To further explore the $F_T/F_C$ dependency on cutting edge radius, experimental work involving measurement of the cutting edge geometries from the work of Yeager et al. (2008) was conducted and is presented below.

**Simulation Results ($h = 0.05$ and $0.2\,\text{mm}$)**

Figure 9 shows specific cutting force and $F_T/F_C$ variations from simulations in terms of material model and at different values of $h$, all with...
$\gamma = +30^\circ$. Cutting edge radius $r_\beta$ was changed from 10 to 20 to 40 $\mu$m as $h$ was changed from 0.05, to 0.1 to 0.2 mm, to keep the ratio $r_\beta/h$ constant. The simulations show no change of ductility (ductile to brittle behaviour) with increasing $h$, i.e., specific forces (and also $F_T/F_C$) are almost independent of $h$. In this respect the model is not realistic, as considered further in the discussion section.

**EXPERIMENTATION**

The carbide inserts used in the work of Yeager et al. (2008) were triangular type TPGN 160308. Different rake angles were created by grinding away the rake face as indicated schematically in Figure 10. They were still available to the present study. The cutting edge profiles of inserts with $\gamma = -30, -10, +10$ and $+30^\circ$, that produced the results shown in Figure 2, were studied in this work in three different ways: by low power optical microscopy ($\approx \times 50$ magnification in the image plane), by white light interferometry and by contact profilometry. In all three cases, the inserts were placed in a V-block, also as shown schematically in Figure 10. Their cutting edges were horizontal and facing upwards. In the two optical studies, therefore, the clearance faces of the inserts were at $\approx 35^\circ$ to the optical axis. The inclinations of the rake face varied with the rake angle.

In contact profilometry the stylus was a $60^\circ$ included angle cone with a $2 \mu$m radius diamond spherical tip. It was traversed across the cutting edge from the flank to the rake face of the tool. For the $\gamma = +30^\circ$ insert, the rake face slope was inclined to the vertical such that once the stylus tip had crossed over the cutting edge the subsequent traverse was between the rake face and the stylus’s conical face.

![FIGURE 10](image) Orientation of insert for edge sharpness measurement (schematic).
RESULTS OF EXPERIMENTS

Figure 11 shows the optical microscopy views of the cutting edge over approximately 1.5 mm length. The edge of the $\gamma = -30^\circ$ insert appears sharp at the magnification shown. In the case of $\gamma = -10^\circ$, the edge is generally sharp, but a small amount of damage is seen. For both the positive rake inserts ($\gamma = +10$ and $+30^\circ$) the edges are seen to be damaged over widths of 50 to 80 $\mu$m. The $\gamma = +30^\circ$ view was taken at the edge of the chip/tool contact area. Portion AB is the as-ground edge and BC is the edge over which cutting has taken place.

Larger magnification views of the cutting edges were obtained by white light interferometry. Figure 12 shows a $\gamma = -30^\circ$ cutting edge, which appears uniformly rounded along the entire length and Figure 13 shows a $\gamma = +30^\circ$ edge at a similar scale. The damage features evident in Figure 11 are seen to be concave scalloped pits potentially attributed to edge fracture. The edge profiles vary from almost flat (Figure 13(b)) to concave (Figure 13(c)) to convex/rounded (Figure 13(d)).

The profiles in Figures 12 and 13 are not at the same magnification horizontally and vertically, which makes a measure of edge radii difficult. Figure 14 shows a small sample of the surface profilometer traces that were made, but now presented with equal horizontal and vertical magnification. Note that in Figure 14 rake faces are to the right whereas in Figures 12 and 13 they are to the left. Figures 14(a) and 14(b) are for a $\gamma = -30^\circ$ insert. A graphical radius gauge is used to measure edge radii, and in these cases radii of 18 and 30 $\mu$m are measured, respectively. Circular edges could be found

FIGURE 11  Optical micrographs of cutting edges from $\gamma = -30^\circ$ (top) to $+30^\circ$ (bottom).
for all the inserts, irrespective of rake angle with range of radius of 25 ± 10 µm. The edge for $\gamma = +30^\circ$ in Figure 14(c) is a typical concave edge that has a damaged length AB of approximately 50 to 60 µm. Although the edge is not distinctly circular, apparent radii can be defined by the inner and outer fitted radius gauges, as shown, with radii of 42 and 64 µm.

**DISCUSSION**

The primary objective of this investigation was to explore the extent to which a metal cutting machining code with a damage accumulation failure
law (and also a pressure dependence of flow stress) could be used to simulate bone machining. Results of the simulations were presented in Figures 4 to 6 over a range of damage and pressure dependence parameters, and adopting a strain hardening plastic flow model with initial yield stress consistent with that reported in the literature. Overall, the responses were comprised of chip formations from continuous to brittle, with specific cutting forces that agreed with those reported for bone machining. However, the specific cutting force was found to be independent of uncut chip thickness (Figure 9(a)) and different damage models were required to match the degrees of ductility shown by chips at different uncut chip thicknesses.

Inspection of Table 2 shows that the three most ductile models, 1 to 3, and the four most brittle ones, 10 to 13, are all models with (in equation 3) high values of $C_p$ (0.5 and 1.0) and similar values of $d_0$ (0.3 and 0.6). It is their pressure sensitivities of yield stress that determine whether they give ductile or brittle behaviour. For the ductile models $a$ (equation 1) = 0.3.

**FIGURE 14** Stylus profilometry traces with graphical radius gauge used to quantify the edge radii: (a) $\gamma = -30^\circ$, $r_b = 18\, \mu m$, (b) $\gamma = -30^\circ$, $r_b = 30\, \mu m$, (c) $\gamma = +30^\circ$, $r_b = 42$ to $64\, \mu m$. 

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**Machining of Cortical Bone**

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whereas for the brittle models $a = 0.6$. In contrast, the intermediate ductility models, 4 to 9, with one exception (model 8), are ones with $C_p = 0$ and for these models the value of $a$ is not so critical (though in most cases $a = 0.3$). In general their brittleness increases as $d_0$ decreases. The intermediate models resulted in segmented chip formations that match the range of behaviour observed experimentally by Wiggins and Malkin (1978). Thus the present work has demonstrated that quantitative models may be created to simulate cortical bone machining. But it has not been explored in depth how the values of $C_p$, $a$ and $d_0$ interact to cause the mechanistic effects that they do.

One of the most important findings from the investigation pertains to the influence of cutting edge radius on the cutting mechanics. The simulations show that the ratio of thrust to cutting force $F_T/F_C$ depends strongly on the edge sharpness (radius) of the cutting tool, with blunter tools having a larger value of $F_T/F_C$ (Figure 8(b)). It leads to a possible explanation of differences in $F_T/F_C$ observed between results of the different researchers for large positive rake angle cutting tools (lined CD, $C/D/\circ$ and $C//D//\circ$ in Figure 2(b)), namely that these researchers used tools with different sharpness and/or tools that quickly underwent a reduction in sharpness during use. Line CD is associated with machining with carbide tools, $C/D/\circ$ with high speed steel tools and $C//D//\circ$ with hardened stainless steel tools. The fracture toughness of these three tool materials is quite different and could lead to different abilities to retain edge sharpness; stainless steel exhibits the highest toughness and should have the best edge retention. The experimental findings (Figures 11 to 14) certainly demonstrate that the edges of the positive rake angle carbide tools broke down sufficiently to explain the high $F_T/F_C$ values reported in Yeager et al. (2008). A practical implication of the findings is that such high rake angles are not suitable for carbide tooling. This is well-known for metal machining and is now seen to apply to machining cortical bone machining as well.

To understand the propensity for edge chipping, supplementary simulations were conducted to estimate the transverse tensile stresses developed at the rake faces of the inserts during cutting of bone, from the instant of first starting to cut up to the development of burr formation at exit. Figure 15 shows results for $\gamma = +30^\circ$ and $h = 0.1$ mm for the work material model of ductility rank 9. The specific cutting force is $\approx 200$ MPa.

The figure is in two parts. The top part shows the variation of cutting and thrust forces with cut distance and the chip formation at four instances as marked, at (a) start of, (b-c) during and (d) at exit from cutting. The bottom part shows the extracted rake face tensile stress distribution at these four instances. At both entry and exit (a and d) thrust forces transiently become negative. They lead to tensile stresses in the rake face that have their maximum value between 0.1 and 0.2 mm from the tip of the
The tensile stresses at exit are larger than those at entry, becoming tensile immediately beyond the edge radius region and reaching a maximum value of 300 MPa. During cutting, transverse stresses are zero.
or compressive, depending on where in the cycle of chip segmentation they are assessed (instances b and c). Highest tensile stresses at exit, associated with burr formation, have been well known for a long time to metal cutting researchers (Pekelharing, 1978). The present results are not new in principal, but they are unique for the application to cutting of bone.

Simulations for the same cutting conditions, except using an insert with $\gamma = -10^\circ$ and $-30^\circ$ showed maximum tensile stresses at exit of 150 and 55 MPa, respectively, and no tensile stresses at entry. As such, more edge breakdown is expected of the positive rake than the negative rake tools. However a tensile stress of 300 MPa would not be expected to cause edge breakdown of a carbide insert. A grade such as C2 (WC 6%Co) has transverse rupture strength in the range 1.5 to 2 GPa (Childs et al., 2000). Of equal concern, Figures 11 to 14 indicate that breakdown of the insert edges was within 0.05 to 0.1 mm of the insert tip, closer than expected from Figure 15 predictions.

More near-exit condition simulation results are shown in Figure 16, for $\gamma = +30^\circ$, $h = 0.1$ mm but for bone material with a larger degree of ductility (rank 2). In this case specific cutting force is approximately 450 MPa (small

FIGURE 16 Cutting and thrust forces, chip formations and tool rake transverse stress distributions at two near exit points (a) and (b): material model ductility rank 2, $\gamma = +30^\circ$, $h = 0.1$ mm.
differences in force and chip formation from the results in Figure 4a for this material arise from the finer mesh used for the Figure 15 results). As with the results of Figure 15 thrust forces become negative over a small cutting distance as burr formation starts (they are associated with a reversal of sliding direction of the chip over the tool). The transverse stress distribution on the rake face at the instant marked (a) is similar to condition (d) in Figure 15. But a more severely stressed state has been captured at the instant marked (b). The tensile stress has risen to a maximum value of 500 MPa and does occur at 0.05 from the cutting tip.

Although this largest tensile stress is greater than the 350 MPa observed in Figure 15, it is not 2.25 times larger as might be expected from the ratio of specific cutting forces in the two cases (450 to 200 MPa). However inspection of Figure 16 shows that at the instant (a) specific cutting force is no longer 450 MPa but has reduced to between 200 and 300 MPa. The magnitude of tensile stress at exit depends in detail on how the changed flow associated with burr formation develops.

Simulations with other material models, keeping the low strain-rate initial yield stress (equation 5) equal to 200 MPa, have not yielded rake face tensile stresses >500 MPa. Even if this value is doubled, in consideration of the observation that the largest experimental specific cutting force recorded in Figure 2a for $\gamma = +30^\circ$ is 800 MPa (at $h = 0.05$ mm), i.e., approximately twice as large as predicted by the largest simulated value, this is still not large enough to explain the edge failure of the carbide inserts. It is necessary to postulate some mechanism for further doubling (to 2000 MPa) the tensile stresses near the cutting edge. One possibility arises from the inhomogeneous structure of cortical bone, with the osteons being harder to cut through than their surrounding matrix. Human and bovine osteon diameters have been reported in the ranges 200 to 300 $\mu$m (Black et al., 1974) and 100 to 250 $\mu$m (Zedda et al., 2008) respectively.

These sizes are similar to those of the individual scalloped pit features of the damaged tool edges in Figures 11 and 13. If local tensile stresses of 2000 MPa were to occur periodically along a cutting edge, where the edge contacts the osteons, these could be sufficient both to cause edge breakdown as observed and to lead to average stresses along the edge of 1000 MPa that could be predicted to be associated with specific forces of 800 MPa. If this explanation is correct, then it could be argued to be an achievement of the modelling and simulations. However, another view is that the essentially homogeneous nature of the material model makes it inherently impossible to use it positively to predict the absolute size of the tensile stresses.

This investigation was the first of its type to explore the application of finite element analysis for understanding the mechanics of chip formation and material removal in machining of bone. It has resulted in new insight
towards understanding previously published results, above those obtained from fracture mechanics based approaches. However there are some obvious limitations that are important to address. For instance, the work concentrates on simulating cutting with the osteons oriented perpendicular to both the cutting edge and the cutting direction. Bone is highly anisotropic, whereas the machining model does not account for that quality. Separate models would need to be developed for evaluating machining in the different orientations. Further, machining of cancellous bone (spongy bone, without fibre re-enforcement) is not considered. Although the feathery chips formed in high speed milling (Jackson et al., 2005) might be able to be modelled as in the present work, that at present has not been tested.

Lastly, some preliminary comments may be made about cut surface temperatures from the simulations. Temperature is one of the most important concerns in machining of bone. It must be maintained at an adequately low temperature to avoid necrosis, typically less than $\approx 55^\circ C$, as mentioned earlier. Figures 3 to 6 show the temperature fields associated with their example chip formations. Chip temperatures commonly exceed $85^\circ C$ and the cut surfaces $55^\circ C$, and that is from an ambient base of $20^\circ C$. A complete collection of results from the simulations indicates that the maximum temperature rise in the cut surface is proportional to the specific cutting force, with the constant of proportionality in the range 0.125 to 0.25 $^\circ C$/MPa. A preliminary conclusion is that, if the cut surface temperature rise in machining cortical bone in vivo is completely (i.e., even at the shallowest depth) to be kept less than approximately $20^\circ C$ (so that the actual temperature is $< 55^\circ C$, taking the ambient temperature of living bone $\approx 37^\circ C$), the specific cutting force should be kept below the range 80 to 160 MPa. These are very low values but not out of line with recent experimental observations from the high speed milling of cortical bone (Sugita et al., 2009b). Also, they are values obtained in the absence of water cooling that is an essential element of surgical practice. Although the temperatures at the cutting edge may not be much reduced by coolant, those further from the edge will be. A more detailed simulation study of temperatures in bone machining, including the effects of water cooling, is essential and will be the topic of further work.

**CONCLUSIONS**

A range of pressure-dependent yield stress models, with strain-path dependent failure laws have been created and used to evaluate the chip formation mechanics in orthogonal cutting of bone. Using an initial yield stress at zero pressure and low strain-rate of 200 MPa resulted in specific cutting forces and chip formations that span those from experimental
studies on the machining of fresh (hydrated) cortical bone. However, the predictions of any one model were independent of uncut chip thickness, which is inconsistent with earlier experimental observations. Different material models need to be selected appropriate for the degrees of ductility observed in practice for any one experimental condition.

One of the primary findings is that the ratio of thrust force to cutting force is sensitive to the cutting edge profile. Thus, differences in force ratio between previously published investigations on machining bone with high positive rake carbide, high speed steel and hardened stainless steel tools, could be attributed to the greater edge chipping, particularly of carbide relative to the other tools. Further simulations provided estimates for the rake face transverse tensile stresses, which were only half the size needed for the rupture of the carbide tools. To fully explain failure of the carbide edges in machining of bone it is necessary to argue that the tensile stresses are concentrated at the contacts of the cutting edge with the osteons within the bone matrix. Further simulations based on a heterogeneous material model would be required to fully explain the tool edge failure behaviour that contributed to the findings of the previously reported experimental investigations.

ACKNOWLEDGMENTS

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NOMENCLATURE

\( F_C, F_T \) cutting and thrust force
\( h \) uncut chip thickness
\( v_c \) cutting speed
\( \gamma \) rake angle
\( \lambda \) resultant force direction
\( \mu \) friction coefficient
\( p \) hydrostatic pressure
\( \sigma \) equivalent flow stress
\( \bar{\varepsilon} \) equivalent strain
\( \dot{\varepsilon} \) equivalent strain rate
\( T \) temperature
\( a, m, n, \varepsilon_0, \sigma_0 \) coefficients in material flow stress laws (equations 1, 5)
\( D \) damage fraction (equation 2)
\( \tilde{\varepsilon}_{f,i} \) equivalent strain to failure in stress state \( i \)
\( C_p, d_i \) coefficients in damage law (equation 3)
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