Estimating the fatigue stress concentration factor of machined surfaces

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Abstract

In this study the effects of surface texture on the fatigue life of a high-strength low-alloy steel were evaluated in terms of the apparent fatigue stress concentration. An abrasive waterjet was used to machine uniaxial dogbone fatigue specimens with specific surface quality from a rolled sheet of AISI 4130 CR steel. The surface texture resulting from machining was characterized using contact profilometry and the surface roughness parameters were used in estimating effective stress concentration factors using the Neuber rule and Arola–Ramulu model. The steel specimens were subjected to tension–tension axial fatigue to failure and changes in the fatigue strength resulting from the surface texture were assessed throughout the stress–life regime (10^7 ≤ N_f ≤ 10^8 cycles). It was found that the fatigue life of AISI 4130 is surface-texture-dependent and that the fatigue strength decreased with an increase in surface roughness. The fatigue stress concentration factor (K_f) of the machined surfaces determined from experiments was found to range from 1.01 to 1.08. Predictions for the effective fatigue stress concentration (K_f) using the Arola–Ramulu model were within 2% of the apparent fatigue stress concentration factors estimated from experimental results. © 2002 Published by Elsevier Science Ltd.

Keywords: Fatigue; Surface roughness; Stress concentration

1. Introduction

Surface roughness and surface integrity resulting from manufacturing processes are both important considerations in fatigue design. In fact, the effects of surface on the fatigue life of metals have been recognized for many years [1,2]. Fatigue damage on the surface of a component typically develops due to the surface integrity resulting from manufacturing, and the presence of stress concentrations originating from the surface topology. In general, the fatigue strength of engineering components increases with a decrease in the surface roughness [3]. However, standard surface roughness parameters cannot always be used for a reliable measure of the reduction in fatigue strength attributed to surface texture [4–6].

In this study the influence of surface texture resulting from machining on the fatigue life of a high-strength low-alloy steel was examined in terms of the apparent stress concentration. The effective fatigue stress concentration factors of the machined surfaces were determined using the Neuber rule [7] and Arola–Ramulu model [8]. The actual fatigue notch factors for the machined surfaces were estimated using experimental results from tension–tension fatigue tests. Through a comparison of the effective stress concentration factors with experimental results, a quantitative evaluation of these parameters is presented and their application to design and fatigue life predictions is discussed.

2. Background

The fatigue strength of a metal is generally defined in terms of the endurance limit, and the effects of surface integrity and surface roughness are considered by correcting the endurance limit by the appropriate factor (k). The surface correction factor (e.g., K_a) is often rep-
represented in terms of the average roughness \((R_a)\), peak-to-valley height roughness \((R_y)\) or 10-point roughness \((R_z)\) of the component surface topography. These parameters are defined in terms of the profile height distribution \((z)\) recorded over an assessment length \((L)\) according to

\[
R_a = \frac{1}{L} \int_0^L [z] \, dx \quad (1a)
\]

\[
R_y = |z_{\text{max}} - z_{\text{min}}| \quad (1b)
\]

\[
R_z = \frac{1}{5} \left[ \sum_{j=1}^5 (z_j)_{\text{max}} - \sum_{j=1}^5 (z_j)_{\text{min}} \right] \quad (1c)
\]

A schematic description of these parameters for an arbitrary machined surface is shown in Fig. 1(a). Note that \(R_a\) describes the average deviation in surface height from the profile mean line and \(R_y\) represents the height from the maximum peak to the lowest valley. The parameter \(R_z\) quantifies the average height from the five highest peaks and five lowest valleys of a surface. All three of the aforementioned parameters are relatively insensitive to specific features of the surface height distribution that are important to fatigue life. For example, the sawtooth and the sinusoidal surface profiles for the turned surfaces in Fig. 1(b) have the same \(R_a\), \(R_y\) and \(R_z\) (assuming that both profiles have equal height amplitudes). Although the sawtooth profile would be much more detrimental to fatigue life by virtue of the small profile valley radii \((\rho)\), the correction factor \((k_a)\) for both profiles in Fig. 1(b) would be equal if defined in terms of a single surface roughness parameter. Standard surface roughness parameters provide a simple and useful means of quantifying profile height distributions, but they should not be used on an individual basis for estimating fatigue strength.

The effects of macroscopic geometric discontinuities on the strength of engineering components are usually approached in terms of the stress concentration factor \((K_t)\). Therefore, it is reasonable for the relationship between surface topography and fatigue strength of metals to be expressed in terms of \(K_t\). The stress concentration factor for a single surface notch in a panel subjected to uniform tension can be described in terms of the notch height \((t)\) and notch root radius \((\rho)\) according to [9]

\[
K_t = 1 + 2\sqrt{\frac{t}{\rho}} \quad (2)
\]

Neuber [7] recognized that the average notch height of surface texture is rarely measured in practice and argued that the features of surface topography are more indicative of successive adjacent notches that promote a lower degree of stress concentration than that for a single notch (Fig. 2). Therefore, Neuber proposed a semi-empirical relationship for the surface stress concentration factor using standard roughness parameters as [7]

\[
K_t = 1 + n\sqrt{\frac{R_z}{\rho}} \quad (3)
\]

where \(R_z\) and \(\rho\) are the 10-point surface height and notch root radius, respectively. The stress state is represented by the factor \(n\) \((n = 1\) for shear and \(n = 2\) for tension) and \(\lambda\) refers to the ratio between spacing and height of

![Fig. 1. The machined surface texture: (a) arbitrary surface profile and the standard surface roughness parameters; (b) idealized sawtooth and sinusoidal machined surface profiles.](image)

![Fig. 2. Near-notch stress trajectories for single and multiple surface notches.](image)
surface irregularities. With reference to the profile in Fig. 2, \( \lambda = b/t \). In practice \( \lambda \) is generally difficult to define for machined surface texture and it has been suggested that \( \lambda = 1 \) is an appropriate choice for mechanically processed surfaces [3].

An alternative expression for the stress concentration imposed by surface texture was recently proposed and used in an evaluation of the effects from net-shape machining on the strength of fiber-reinforced plastics (FRPs) [10]. The effective stress concentration \( (K_e) \) for the process-dependent surface texture was defined in terms of dominant profile valleys and the corresponding average valley radii. The final expression for \( K_e \) expressed in terms of standard roughness parameters is given by

\[
K_e = 1 + n\left(\frac{R_y}{\bar{R}}\right)\left(\frac{R_y}{R_z}\right)
\]

(4)

where \( R_y, R_z \), and \( \bar{R} \) are the average roughness, peak-to-valley height and 10-point roughness as described in Eqs. (1). The parameter \( \bar{R} \) is the effective profile valley radius and represents the average radius determined from the dominant profile valleys. Material dependencies and load type effects are accounted for through the empirical constant \( (n) \) for convenience. Similar to the convention used for the Neuber rule, \( n = 2 \) for uniform tension and \( n = 1 \) for shear loads. Although validated for the effects of machined surface texture on the strength of FRPs under static and dynamic loads [10], no effort has been made to examine the use of Eq. (4) for estimating the fatigue strength of metals.

If the effective stress concentration factor for surface roughness is obtained from Eq. (3) or Eq. (4), the effective fatigue stress concentration \( (K_{ef}) \) can be obtained from \( K_e \) according to [9]

\[
K_{ef} = 1 + q(K_e - 1)
\]

(5)

The notch sensitivity \( (q) \) can be defined in terms of the effective profile valley radius of the surface texture \( \bar{R} \) in place of the single notch root radius \( \bar{\rho} \) according to [9]

\[
q = \frac{1}{(1 + \gamma/\bar{\rho})}
\]

(6)

where \( \gamma \) is a material constant. For steels, \( \gamma \) is defined in terms of the ultimate strength \( (\sigma_u) \) as [11]

\[
\gamma = 0.025\left(\frac{2070 \text{ MPa}}{\sigma_u}\right)^{1.8} \text{ mm} \quad (\sigma_u \geq 550 \text{ MPa})
\]

(7)

3. Materials and methods

The effects of surface texture on fatigue strength were evaluated using AISI 4130 CR steel. The material was obtained in sheet form with 3.2 mm thickness and exhibited a yield and ultimate strength of 655 and 752 MPa, respectively.

An Omax model 2652 abrasive waterjet (AWJ) was used to machine fatigue specimens from the AISI 4130 sheet. The conventional dogbone fatigue specimen geometry was adopted with a gage section width of 12.5 mm and remaining dimensions defined in accordance with ASTM E466-82. All specimens were machined such that the axis of tensile loading was parallel to the rolling direction of the AISI 4130 sheet. A design matrix for fatigue testing was established using specimens with three separate surface qualities including an average roughness of 2 \( \mu \)m (AWJ A), 4 \( \mu \)m (AWJ B) and 6 \( \mu \)m (AWJ C). A total of 56 specimens were machined with each surface quality using the cutting parameters listed in Table 1. Fifteen additional specimens were machined using the conditions for AWJ A (Table 1) and then carefully polished using #220 and #400 grit sandpaper and emery cloth, in succession, to achieve an average roughness of less than 0.1 \( \mu \)m. These specimens served as the control (with benchmark quality) and were used to determine the fatigue strength of the AISI 4130 CR steel. The residual stress in all of the AWJ machined surfaces was assumed to be compressive, and of approximately the same magnitude, regardless of cutting conditions [12]. Hence, the effects of the surface integrity on the fatigue life of the AISI 4130 were assumed to be identical regardless of differences in the cutting conditions.

A Hommel T8000 contact profilometer was used to analyze the texture resulting from AWJ machining using a skid-type probe with diamond stylus of 10 \( \mu \)m diameter. The average surface roughness \( (R_y) \), 10-point roughness \( (R_z) \) and peak-to-valley height \( (R_m) \) were calculated according to ANSI B46.1 using a traverse length of 4.8 mm and a cutoff length of 0.8 mm. The effective notch root radius \( (\bar{\rho}) \) was estimated from the surface profiles using a graphical radius gage [8,10]. A best-fit circle defined by the maximum area of contact was inscribed in the root of at least three critical valleys of each profile examined. The average of these profile valley radii estimated from selected surface profiles was then calculated to establish \( \bar{\rho} \) for each specific group of machined specimens (e.g., AWJ A, etc.).

The machined specimens were subjected to constant-amplitude tension–tension axial fatigue using an MTS 810 tension/torsion frame. Eight specimens were tested at seven load levels that spanned the expected stress–life fatigue response \((8 \times 7 = 56 \text{ specimens of each surface quality); the maximum axial stress (} \sigma_0 \text{) for the seven load levels ranged from 199 to 333 MPa. All fatigue tests were conducted in load control with a sinusoidal load waveform, frequency of 12 Hz and a stress ratio (} R \text{) of 0.1. The load and displacement history were recorded in intervals over the fatigue response of each specimen using Teststar II control software which
accompanies the MTS. Following failure of each specimen, the number of cycles to failure \(N_f\) was recorded and the fractured surface was inspected to identify the origin of failure. The control specimens \(R_a \leq 0.1 \mu m\) were tested using the staircase method [13] to determine the endurance limit of the AISI 4130 CR steel. The fatigue limit for both the machined and the control specimens was defined at \(N_f = 10^6\) cycles.

A statistical analysis of the fatigue life for the AWJ machined steel was conducted using Weibull statistics [14]. The probability density distribution of the fatigue response obtained for each surface quality and load level was adequately described using a two-parameter Weibull distribution. A 95% confidence interval was used to identify outliers and acquire a censored fatigue response at each load level. A Student’s \(t\)-test was then performed \((\alpha = 0.05)\) with the cumulative results at each stress level to establish the statistical significance of observed differences in fatigue life attributed to the machined surface quality. The fatigue life distribution resulting from specimens of each surface quality was also used to develop an empirical relationship for the fatigue life over the stress–life regime \((10^3 \leq N_f \leq 10^6\) cycles\). The fatigue strength (endurance limit) for the AISI 4130 with each surface quality was then determined from the empirical relationships by solving for the cyclic stress amplitude resulting in a fatigue life of \(N_f = 10^6\) cycles.

### 4. Results

Surface profiles of the AWJ machined AISI 4130 CR steel were recorded and used to determine the roughness and profile valley radii of each machined surface quality. Typical profiles from specimens with 2 and 6 \(\mu m\) average roughness are shown in Fig. 3(a) and (b), respectively. Examples of profile valley radii for these two surfaces are shown in Fig. 3(c) and (d). The standard surface roughness parameters and effective profile valley radii \(\bar{R}_v\) for each of the four groups of specimens are listed in Table 2. Profile valley radii of the control specimens were not available, as the surface did not exhibit prominent valleys after polishing was completed. Although the AWJ machined surface with smallest surface roughness (AWJ A) also exhibited the largest effective valley radius, differences in the profile valley radii resulting from the three cutting conditions were limited.

Fig. 3. Surface texture and profile valley radii of the AWJ machined surfaces: (a) surface profile with 2 \(\mu m\) \(R_a\); (b) surface profile with 6 \(\mu m\) \(R_a\); (c) profile valley radius from (a); (d) profile valley radius from (b).

The fatigue strength of the AISI 4130 CR steel was determined using the staircase method under zero to tensile loading with specimens of average surface roughness equal to 0.07 \(\mu m\). The strength was found to be 225 MPa using a fatigue limit defined at \(N_f = 10^6\) cycles. The fatigue life diagram representing all results (uncensored) from fatigue testing of the AWJ machined specimens is shown in Fig. 4. As evident in this figure, there was a notable difference in fatigue strength amongst the three groups of specimens. At low stress levels, or under high-cycle fatigue (HCF), the fatigue strength decreased with an increase in \(R_a\). As expected, the specimens with the lowest roughness (AWJ A) exhibited superior fatigue strength while specimens with the highest surface roughness (AWJ C) exhibited the lowest fatigue strength. In contrast, at high stress amplitudes or low-cycle fatigue (LCF), the fatigue strength increased with an increase in \(R_a\). The transition in fatigue response of the AISI 4130 CR steel from LCF to HCF appears to occur near a stress amplitude of

### Table 1
Cutting conditions used for AWJ machining of fatigue specimens

<table>
<thead>
<tr>
<th>(R_a) ((\mu m))</th>
<th>Pressure (MPa)</th>
<th>Transverse speed (mm/s)</th>
<th>Standoff (mm)</th>
<th>Grit size (garnet#)</th>
<th>Abrasive flow rate (g/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>310</td>
<td>1.8</td>
<td>0.8</td>
<td>120a</td>
<td>4.9</td>
</tr>
<tr>
<td>2</td>
<td>310</td>
<td>1.8</td>
<td>0.8</td>
<td>120</td>
<td>4.9</td>
</tr>
<tr>
<td>4</td>
<td>310</td>
<td>2.1</td>
<td>0.8</td>
<td>80</td>
<td>4.5</td>
</tr>
<tr>
<td>6</td>
<td>310</td>
<td>5.5</td>
<td>0.8</td>
<td>50</td>
<td>2.3</td>
</tr>
</tbody>
</table>

*a Followed by polishing with sandpaper and emery cloth.
Table 2
Surface roughness and notch sensitivity of the machined surfaces

<table>
<thead>
<tr>
<th>Method</th>
<th>$R_a$ (µm)</th>
<th>$R_y$ (µm)</th>
<th>$R_z$ (µm)</th>
<th>$\bar{\rho}$ (µm)</th>
<th>$q$ [Eq. (6)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>0.07</td>
<td>0.71</td>
<td>0.74</td>
<td>N/A</td>
<td>1.00</td>
</tr>
<tr>
<td>AJW A</td>
<td>1.96</td>
<td>12.70</td>
<td>13.19</td>
<td>10.80</td>
<td>0.067</td>
</tr>
<tr>
<td>AJW B</td>
<td>3.91</td>
<td>22.08</td>
<td>22.67</td>
<td>9.20</td>
<td>0.058</td>
</tr>
<tr>
<td>AJW C</td>
<td>6.04</td>
<td>29.07</td>
<td>30.71</td>
<td>9.00</td>
<td>0.057</td>
</tr>
</tbody>
</table>

Fig. 4. Fatigue life diagram of the AWJ machined AISI 4130 CR steel (uncensored data including results from all specimens).

approximately 315 MPa ($N = 10^4$ cycles) as evident in Fig. 4. The AWJ C specimens showed superior fatigue strength under low-cycle fatigue ($N \leq 10^4$ cycles) and inferior fatigue strength relative to the specimens with low roughness (AJW A) under high-cycle fatigue ($N > 10^4$ cycles).

An examination of the fractured AISI 4130 CR steel specimens using optical microscopy revealed that features of specimens that failed under low- and high-cycle fatigue were quite different (Fig. 5). Considerable evidence of plastic deformation was apparent from the necking and cup–cone fracture surface of the LCF specimens ($N \leq 10^4$ cycles), whereas the HCF specimens showed little or no evidence of plastic deformation. For all AWJ machined specimens that failed at $N > 10^5$ cycles (HCF), the origin of failure was clearly evident at the machined surface and a finite portion of the fractured surface was oriented normal to the applied cyclic tensile stress with clear indication of crack initiation and cyclic growth. The second portion of fracture surface area exhibited a rough texture and prominent shear lip inclined 45° to the applied load. An example of these features from the surface of a fractured specimen is shown in Fig. 5(a). Therefore, under HCF, the dominant source of specimen failure was fatigue crack initiation from discontinuities existing at the machined surface followed by crack propagation until the flaw reached a critical length. The LCF steel specimens exhibited necking and the entire fractured surface exhibited a shear lip as shown in Fig. 5(b). Thus, fatigue damage within the LCF specimens appeared to originate primarily through the coalescence of internal defects; the machined surface integrity contributed to damage accumulation but was of secondary importance. Fatigue failure of the steel specimens tested within the LCF region occurred through ductile fracture.

5. Discussion

Results from fatigue testing of the AWJ machined AISI 4130 CR steel indicated that the magnitude of surface roughness affected the material’s fatigue strength. A reduction in fatigue life occurred with increasing roughness of the steel specimens under HCF and undoubtedly occurred due to the stress concentration posed by the surface roughness. However, the low-cycle fatigue strength increased with increasing surface rough-
ness and was not expected. As evident in Fig. 4, the fatigue life diagram obtained from experimental results exhibits a transition in the fatigue process at approximately $N = 10^6$ cycles and clearly demarcates the boundary between LCF and HCF. Results from the Student’s $t$-test indicated that the difference in fatigue strength between the AWJ A and AWJ C specimens within both the LCF and HCF was statistically significant at the $\alpha = 0.05$ level. Thus, the changes in fatigue strength with surface roughness observed in both low- and high-cycle fatigue regions were not a reflection of statistical scatter.

Differences in the fracture surfaces of low- and high-cycle fatigue specimens (Fig. 5) distinguished the changes in fatigue failure attributed to surface roughness within the LCF and HCF regions of testing. Microscopic observations clearly indicated that fatigue failure of the HCF specimens initiated at surface defects that resulted from machining, regardless of the magnitude of surface roughness. The coarse fracture surface and prominent necking of the LCF specimens [Fig. 5(b)] suggested that failure occurred primarily through the coalescence of internal defects. Although external defects and the machined surface integrity continued to contribute to fatigue damage accumulation and the failure process in low-cycle fatigue, no differences in the fracture surfaces were apparent between the three groups of steel specimens. Nevertheless, results from fatigue testing indicated that there was a significant difference in the LCF life between the specimens with 2 $\mu m$ and 6 $\mu m$ average roughness. In some manner the rate of damage accumulation and corresponding contribution of surface integrity to fatigue failure of the AWJ C specimens with largest roughness were suppressed. This may have resulted at the high stress amplitudes from near-surface plastic deformation and the reverse yielding phenomenon [15]. According to this argument, the increase in low-cycle deformation and the reverse yielding phenomenon [15]. The effective stress concentration factors ($K_\alpha$) for the AWJ machined surfaces were calculated using the Neuber rule and Arola–Ramulu model according to Eqs. (3) and (4), respectively, and are shown in terms of the surface roughness in Fig. 7(a). A load factor ($n$) of 2 was used for both models to account for the uniform tensile loads. Note the significant difference in stress concentration predicted from the two models. Using the notch sensitivity for each surface texture as indicated in Fig. 6 and listed in Table 2, the effective fatigue stress concentration ($K_\alpha$) predicted from the Neuber rule and the Arola–Ramulu model are shown in Fig. 7(b). The apparent fatigue stress concentration factor ($K_\alpha$) was also determined from experimental results in terms of the ratio of the fatigue strength of each surface quality to the fatigue strength of the control specimens at a fatigue life of $10^6$ cycles. It is apparent from Fig. 7(b) that predictions for $K_\alpha$ obtained from the Arola–Ramulu model

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{Relationship between the average surface roughness ($R_a$) and notch sensitivity ($q$).}
\end{figure}
are more consistent with the fatigue response for the AISI 4130 CR steel. The $K_f$ predicted using the Arola–Ramulu model was within 2% of the apparent fatigue stress concentration estimated from experimental results. Therefore, $K_f$ may serve as a useful tool to account for the effects of surface texture on the fatigue strength of engineering materials.

In this investigation the Arola–Ramulu model was used to calculate $K_t$ and the corresponding $K_f$ for the AWJ machined surfaces. The $K_t$ of each surface was then compared to $K_f$ that was determined at $N_f = 10^6$ cycles using experimental results. Results from fatigue testing of the control specimens using the stair-case method indicated that the endurance limit of the AISI 4130 CR steel was very near $10^6$ cycles. Thus, the comparison of $K_t$ and $K_f$ conducted in this study was essentially a comparison of the endurance strengths. If $K_f$ is estimated from the machined surface of an engineering component it should be used to estimate the corrected endurance strength. Then the fatigue strength could be estimated at any finite life within the stress life regime ($10^3 \leq N \leq 10^6$ cycles) using the desired approach (e.g., Juvinall, etc). Based on results of this study performed with a stress ratio of 0.1, it would appear that predictions for fatigue strength using this method would become increasingly conservative with shorter finite life. The same may not be true at other cyclic stress ratios, especially those with substantial mean stress. These concerns should be addressed and will be the focus of our future research.

6. Conclusion

Based on the results from the surface texture evaluation and axial tension–tension fatigue tests conducted with AISI 4130 CR steel, the following conclusions were drawn.

1. The axial tension–tension fatigue life of AISI 4130 CR steel is surface-texture-dependent. As the surface roughness resulting from AWJ machining increased from 2 to 6 µm, the high-cycle fatigue life decreased. However, an increase in fatigue life occurred with increasing surface roughness under low-cycle fatigue and resulted from the surface stress concentration and contribution of damage retardation through near-notch yielding.

2. The notch sensitivity of the AWJ machined steel specimens did not change significantly with increase in surface roughness. Average surface roughness should not be used in interpreting the apparent notch sensitivity of a machined surface since it does not account for the process-dependent profile valley radii.

3. The effective elastic stress concentration factor ($K_t$) and corresponding fatigue stress concentration factor ($K_f$) for the machined surfaces were calculated using the Neuber rule and Arola–Ramulu model. The maximum error between the apparent fatigue stress concentration from experiments and $K_f$ determined using the Arola–Ramulu model was 2%.

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